Modelling of the Rhine region of freshwater influence

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Abstract

In order to provide pilots guiding ships near the harbour of Rotterdam in the Netherlands with accurate current information a considerable effort has gone into the modelling of the region around the 'Hook of Holland'.

The currents in this region to a large extend are influenced by the flow of fresh water from the Meuse and Rhine rivers and by the wind. This necessitates the on-line forecasting of the currents, since the number of possible combinations of wind and river outflow is simply too large to tabulate. Moreover, the clear three dimensional structure of the currents makes the use of a three dimensional model necessary.

In the present study a three dimensional model of the area was compared with a large set of measurements for various settings of the model parameters and using several turbulence models. The measurements include HF-radar measurements of surface currents and conventinal current measurements. The results indicate that wind and stratification play an important role and that these effects can be modelled reasonably well using a three dimensional model.

Future research will aim at assimilating HF-radar surface current measurements into the three dimensional model presented here using an efficient approximation of the time-varying Kalman filter. Some experiments were performed to asses the feasibility of this procedure.

1 Introduction

The Rotterdam Europort harbour is one of the busiest shipping areas in the world. The sizes of the ships in this area is very mixed and ranges from 10 meter yachts to huge container carriers. The largest ships are confined to the euro channel and Meuse channel and need a depth of 20 meters. This means that they can not enter for some hours during low tide. The time window in which they can is influenced by negative storm surges caused by seaward wind and by the height of the waves.

Besides the depth of the water also the current is important. The large ships can only manoeuvre slowly and can only react to changes in the current with difficulty. Therefore, pilots are used assist and avoid dangerous situations. If necessary, tug-boats are used for additional manouvrability.

Usually pilots learn the typical patterns of currents and dangerous spots by experience. However, the flow in this region is complex and may vary strongly due to wind and river discharge. In addition new constructions, like dams and channels may alter the pattern.

To assist the pilots a current atlas is provided, in which the typical currents are shown for one tidal cycle. These charts are based on measured currents in which the effects of neap tide and spring tide are averaged and only one or two typical Rhine outflows are considered. The measurements are carried out in calm weather to reduce the meteorological effect in the resulting currents.

Some attempts have been made to produce the current atlas using one dimensional and two dimensional models. In principle the modelling approach is less costly and the meteorological effects can be fully eliminated. Mainly due to the strong three dimensional structure these attempts have not been successful. Therefore, three dimensional models were constructed and have become available recently. Another advantage of models is that also forecasts that include meteorological effects can be made. This information can be used to construct an electronic current atlas.

A parallel development is the use of advanced measurement equipment, such as Acoustic Doppler Current Profilers (or ADCP's) that measure the current profile at a certain point almost continuously and automatically transmit the information to the shore. A disadvantage is that measurement equipment can not be positioned in the channel, since this could result in damage to ships or the equipment. Thus a representative position outside the channel has to be chosen with currents that are highly correlated to the currents in the channel.

Surface currents can also be measured with a special kind of radar, HF-radar. A great advantage is that one can measure also in the relevant areas, since the equipment is all positioned ashore. Unfortunately, the radar signal does not penetrate further than about one meter into the water, while for guiding large ships the current averaged over the upper 20 meters is needed.

Moreover, the surface currents can not be extrapolated downwards using a one dimensional model (in the vertical direction) because of the complex dynamics in this area. It was observed in the 'Nieuwe Waterweg' that during part of the tidal cycle the surface current and the current near the bottom have opposite directions .

A third approach is to integrate a three dimensional model, ADCP current profile measurements and HF-radar measurements of the surface currents. The information from these three sources is combined using a data assimilation algorithm. In this way the model is kept 'on track' by the measurements and the measurements are interpolated (or extrapolated) in space and time using the model. Data assimilation has been used successfully in meteorology, oceanography, hydrology and many other areas. The weather forecasts in many countries are based on an integration of model and measurements, i.e. data assimilation.

In the present study, the results of a three dimensional model are compared with conventional current measurements and HF-radar surface currents. The aim is to assess the accuracy of both and to determine whether data assimilation is feasible.

2 Model

The flow in rivers, sea's and estuaries can be modelled quite accurately by the three dimensional shallow water equations. These equations can be deried from the well known Navier-Stokes equation by assuming hydrostatic pressure, using the Boussinesc approximation and Reynolds averaging in time. This results in the following equations

$$\frac{\partial \xi}{\partial t} + \frac{\partial(\int_{-d}^{\xi} u dz)}{\partial x} + \frac{\partial(\int_{-d}^{\xi} v dz)}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\frac{\partial p}{\partial x}}{\rho} - fv - \frac{\frac{\partial \tau_x}{\partial z}}{\rho} = 0 \quad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\frac{\partial p}{\partial y}}{\rho} + fu - \frac{\frac{\partial \tau_y}{\partial z}}{\rho} = 0 \quad (3)$$

$$w + \frac{\partial(\int_{-d}^{z} u dz)}{\partial x} + \frac{\partial(\int_{-d}^{z} v dz)}{\partial y} = 0 \quad (4)$$

with

$$p = p_a + g\int_z^{\xi} \rho(s) dz \quad (5)$$

$$\tau_x = \begin{cases} \rho \nu_v \frac{\partial u}{\partial z} & -h < z < \xi \\ c_D \rho_{air} |(u_w, v_w)| u_w & z = \xi \\ \frac{-g}{C^2} |(u,v)| u & z = -h \end{cases} \quad (6)$$

$$\tau_y = \begin{cases} \rho \nu_v \frac{\partial v}{\partial z} & -h < z < \xi \\ c_D \rho_{air} |(u_w, v_w)| v_w & z = \xi \\ \frac{-g}{C^2} |(u, v)| v & z = -h \end{cases} \quad (7)$$

where

variable	description
t	time
(x, y)	position in horizontal plane
z	position above reference plane
ξ	waterlevel above the reference plane
u, v, z	current in x, y and z (=up) directions
p	pressure
s	salinity
ρ	density of the water
f	constant for corriolis
d	depth below reference level
$D = d + \xi$	total water depth
g	acceleration of gravity
τ_x, τ_y	stresses in x and y directions
ν_v	eddy viscosity in vertical direction
ν_h	eddy viscosity in horizontal direction
c_D	drag coefficient in wind stress
u_w, v_w	wind velocity in x and y directions
T	temperature

Equation 1,4 describe the conservation of mass, equations 2 and 3 the conservation of momentum. Equation 5 shows the hydrostatic pressure relation and equations 6 and 7 the computation of stresses between horizontal layers, top and bottom.

If there are significant differences in salinity, like in our application, then the density can not be assumed constant. The following empirical formula's (Eckhart) were used

$$\rho = \frac{P_0}{\lambda + \alpha_0 P_0} \quad (8)$$

$$\lambda = 1779.5 + 11.25T - 0.0745T^2 - (3.80 + 0.01T)s \quad (9)$$

$$\alpha_0 = 0.6980 \quad (10)$$

$$P_0 = 5890 + 38T - 0.37T^2 + 3s \quad (11)$$

To obtain a closed set of equations, an additional transport equation for the salinity is needed

$$\frac{\partial s}{\partial t} + \frac{\partial us}{\partial x} + \frac{\partial vs}{\partial y} + \frac{\partial ws}{\partial z} - \frac{\partial D_h \frac{\partial s}{\partial x}}{\partial x} - \frac{\partial D_h \frac{\partial s}{\partial y}}{\partial y} - \frac{\partial D_v \frac{\partial s}{\partial z}}{\partial z} = 0 \quad (12)$$

where D_h and D_v are the horizontal and vertical eddy diffusivity coefficients. Often the eddy diffusivity is related to the eddy viscosity using the Schmidt number σ.

$$D_h = \nu_h/\sigma_h \quad (13)$$
$$D_v = \nu_v/\sigma_v \quad (14)$$

The turbulence models used for the experiments are both based on the Prandtl mixing length principle (see [6] for an overview). The first turbulence model is algebraic. The turbulent kinetic energy at surface and bottom are derived from the stresses at the surface and the bottom. Linear interpolation is used in between. Finally the mixing length is based on the distance to surface and bottom.

$$L = \kappa(z+d)\sqrt{1-\frac{z+d}{H}} \quad (15)$$
$$k = \frac{1}{\rho c_\mu}\left(\tau(z=-h)(1-\frac{z+d}{H}) + \tau(z=\xi)\frac{z+d}{H}\right) \quad (16)$$
$$\nu_v = c_{\mu'} L\sqrt{k} \quad (17)$$

where

variable	description
k	turbulent kinetic energy
L	mixing length
$\kappa \approx 0.41$	von Karman constant
$c_{\mu'} \approx 0.58$	calibration constant
$c_\mu \approx 0.09$	calibration constant

In more complex flows the direct specification of a mixing length is difficult and it is more convenient to use the dissipation rate of kinetic energy. The $k-\varepsilon$ model is based on this idea and uses advection equations for both the turbulent kinetic energy k and the dissipation rate ε.

$$\frac{\partial k}{\partial t} + \frac{\partial uk}{\partial x} + \frac{\partial vk}{\partial y} + \frac{\partial wk}{\partial z} - \frac{\partial D_h \frac{\partial k}{\partial x}}{\partial x} - \frac{\partial D_h \frac{\partial k}{\partial y}}{\partial y} - \frac{\partial D_v \frac{\partial k}{\partial z}}{\partial z} = P_k + B_k - \varepsilon$$
$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial u\varepsilon}{\partial x} + \frac{\partial v\varepsilon}{\partial y} + \frac{\partial w\varepsilon}{\partial z} - \frac{\partial D_h \frac{\partial k}{\partial x}}{\partial x} - \frac{\partial D_h \frac{\partial k}{\partial y}}{\partial y} - \frac{\partial D_v \frac{\partial k}{\partial z}}{\partial z} = P_\varepsilon + B_\varepsilon - \epsilon_\varepsilon$$
$$\nu_v = C_\mu \frac{k^2}{\varepsilon}$$

where $P_k, P_\varepsilon, \varepsilon, \varepsilon_k$ are production and dissipation terms and the terms B_k and B_ε are damping terms for vertical density gradients.

To complete the model various boundary conditions are needed. Their treatment falls outside the scope of this paper and the interested reader is referred to Leendertse ([6]).

To simulate the model the equations above were discretized. For efficiency a staggered grid was used. In time an Alternating Directions Implicit method for the horizontal plane is combined with a fully implicit solution for the vertical direction. In the horizontal plane curvilinear coordinates were used; in the vertical σ coordinates. A detailed description can be found in [4].

3 Data

Four types of measurements were used for the experiments: waterlevel measurements at some locations near the coast, current measurements at several points and depths along a transect starting near Scheveningen, measured wind velocity and direction at Noordwijk and HF-radar measurements. Figure 1 shows the locations of the various measuring devices.

The HF-radar provides measurements of currents near the surface. It is based on measurement of the Doppler shift of the return signal. From the Doppler shift the radial component of the current can be deduced. Therefore two stations are needed. The radar signal can only penetrate about one meter into the sea-water and thus the return signal in only influenced by currents in the upper meter. The radar signal is transmitted in a narrow beam, the angle of which is constantly adapted in order to scan the whole area. The measurements cover an area of about 25km by 25km with a resolution of 1km. Thus a synoptic picture is produced, that is updated every 30 minutes.

The measurements used in this paper were obtained as a part of the European MAST project in October 1990. The area of interest of this campaign was slightly north of the harbour area, but many of the physical processes are similar in this area.

4 Data Assimilation

At the moment, there is no model that can accurately reproduce the complicated current patterns in area around Hook of Holland under all conditions and thus produce reliable forecasts for guiding ships. The HF-radar can accurately measure the currents in this area, but only for the upper 1 meter of the water column. The aim of this project is to integrate these two sources of information. This way the HF-radar measurements are extrapolated to greater depths and the model forecasts are improved. This process is often called data assimilation, but others may use terms as model inversion, filtering or integration of data and model.

There are several techniques available to solve data assimilation problems. The most popular are Optimal Interpolation, Kriging, Variational methods (adjoint), Kalman filtering and the representer method (some references are [2, 5, 1]).

For the present application we have chosen for Kalman filtering because the estimates can be updated easily to obtain the forecast for the next period. A difficulty was that the 'full' Kalman filter requires far to much computations. Therefore an efficient approximation, the Reduced Rank Square Root (RRSQRT) filter was developed [9, 10]. This suboptimal scheme (see [2] for a definition) reduced the computation times to a small fraction of those for the 'full' Kalman filter.

5 Experiments

To assess the accuracy of the present model, the accuracy of the HF-radar measurements and the possibility of data assimilation a large number of model runs was performed and the results were compared with the various measurements.

The grid of the model used in the experiments is shown in figure 2. The model was deliberately taken quite small since the aim is to use data assimilation for the correction for errors in the external forcing.

For such a small scale model most of the setup, which is caused by wind and pressure gradients, is generated outside the model and should therefore be included into to boundary conditions. In practice the meteorological effect at the open boundary is not known. For this reason usually a large model with a coarser grid is used to compute these effects. In this project we use the measured set up in Hook of Holland to correct the open boundary. After some filtering with a Godin filter [3] this produced accurate results for the waterlevels in the model (see figure 3). This shows that even a very simple data assimilation scheme can eliminate the need for a larger model in case only now-casts and hind-casts are needed.

A second external forcing of the model is the stress caused by wind on the water surface. In larger models the wind-input is often produced by a meteorological model. In this case the measured winds at Noordwijk were used as a global input for the model. The errors of using global time-series for the wind forcing are expected to be small, since the correlation of the measured wind series of Noordwijk and Hook of Holland had a correlation of 0.9 over October '90.

Although the waterlevels can be reproduced quite accurately by the model, it is much more difficult to obtain accurate currents. In long shore direction the depth averaged current is dominated by the tide, which can be modelled quite well. The profile however is strongly influenced by the stratification, due to the inflow of fresh-water from the Rhine and Meuse rivers. The vertical mixing of the fresh-water and salt seawater is in turn influenced by the turbulence generated by the tide and wind and damped by smaller turbulent fluctuations near the salinity gradient.

A very important secondary effect are the cross-shore currents generated by the Corriolis force acting on the differences in longshore currents at different depths (see [8, 7] for a description of this effect, called Tidal Straining). Figure 4 shows some cross-shore current time-series with the algebraic turbulence model and with the $k - \varepsilon$ turbulence model. It is clear that only the $k - \varepsilon$ model can capture the tidal straining effects during periods with large stratification (October 8–12). Before October 8 and after October 12 the wind was so strong that the water was well mixed. Further experiments indicate that the model results are very sensitive to vertical resolution and the various parameters that control the mixing rate. Therefore, some effort will be spent in further adaptation of the parameters in the future.

6 Future research

For the remaining differences between model and measurements we believe the main causes are inaccurate formulation of the wind-stress as a function of the wind velocity and errors in the vertical mixing rate. The first type of errors can be observed from the HF-radar measurements of the surface currents, while errors of the second type can be detected using measurements of the current profile. The use of 'acoustic Doppler' measurements can produce the accurate current profiles needed for this.

The fact that the main errors in the model results can be observed with the HF-radar and profile measurements shows that both sources of information are complementary. It thus is expected that data assimilation can have a significant impact. For this purpose we are currently extending our Kalman filter for the use of these measurements. Much of the experience so far has been with the use of waterlevel measurements. In these data assimilation experiments the results are very promising.

7 Conclusions

The experiments in this paper indicate that a three dimensional shallow water model with a $k - \varepsilon$ turbulence model is able to predict the currents with a reasonable accuracy and this accuracy can probably be improved by further adaptation of the parameters that control the vertical mixing. However, especially in windy conditions the results can probably be improved significantly by assimilating remote sensing measurements of the surface currents. Furthermore by incorporating waterlevel measurements the need for a larger (nested) model to compute the wind set-up at the open boundary is eliminated, so that the system can run with a local model and local measurements only.

An accurate prediction of the currents would greatly improve the possibility of handling ships in rough weather in the area of the Rotterdam harbour. An on-line electronic 'current atlas' could provide the accurate current infomation needed.

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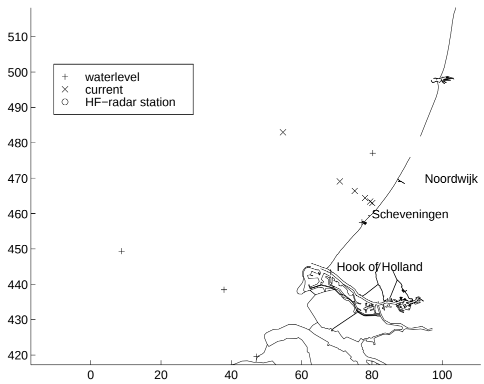

figure 1: Measurement locations

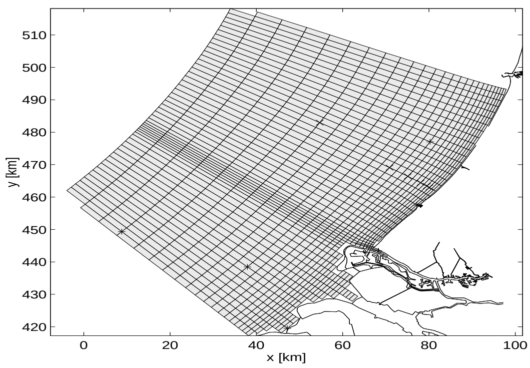

figure 2: Model grid

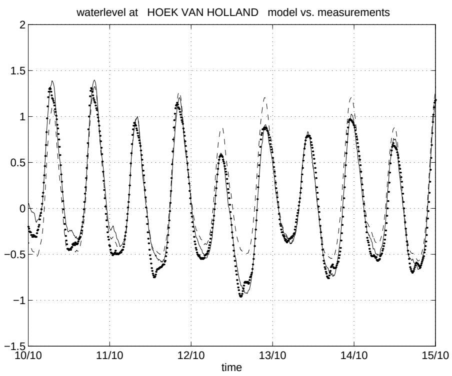

figure 3: Waterlevels with astronomic (dashed) and corrected (line) boundary conditions vs. measurements (dotted)

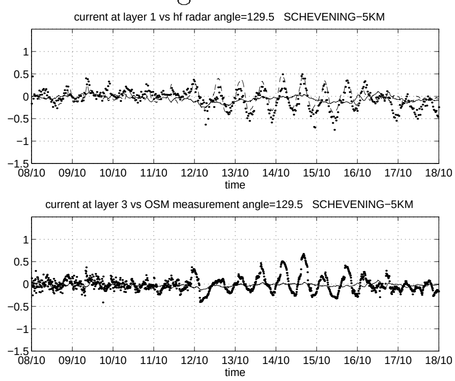

figure 4: Cross-shore currents for algebraic (line) and $k-\varepsilon$ model (dashed) vs. measurements (dotted)

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